ORIGINAL WORKED EXAMPLE

Flitch beam
Elastic stiffness and load sharing

Compare a timber-and-steel beam at a fixed overall depth. Calculate the stiffness, bending-moment share, elastic stress and immediate bending deflection of each option.

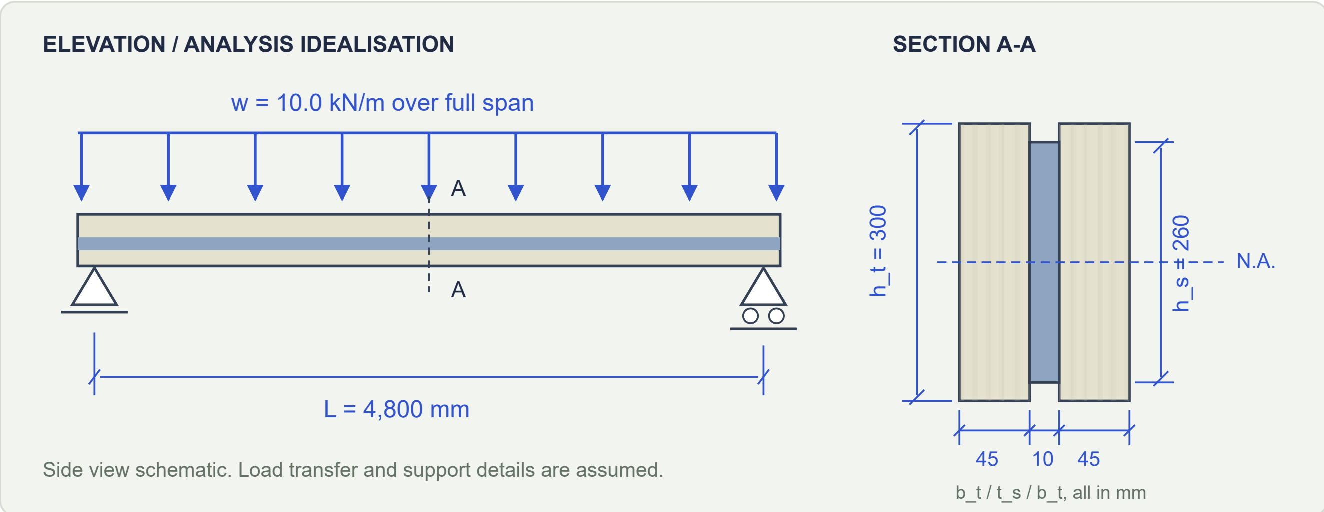


Figure 1. Original analysis schematic. The plate is centred vertically; its depth is independent of the timber depth. The section is diagrammatic, with plate thickness exaggerated.

1. Reference case: FB-03

Input	Symbol	Adopted value	Basis
Support-centre span	L	4,800 mm	Fictional project geometry
Each timber member	$b_t \times h_t$	45 × 300 mm	Two identical members
Central steel plate	$t_s \times h_s$	10 × 260 mm	20 mm inset at top and bottom
Timber elastic modulus	E_t	10,000 MPa	Assumed along-grain modulus
Steel elastic modulus	E_s	200,000 MPa	Assumed elastic modulus
Total uniform service load	w	10.0 kN/m	Adopted total, including self-weight
Bending-deflection target	δ_{lim}	L / 300 = 16.00 mm	Chosen demonstration criterion

Basis of this note. This is an original Fontana demonstration with fictional project inputs and assumed material moduli. It supplies no timber grade, steel strength, connector capacity or mandatory design limit. The equations analyse the idealisation on page 2.

CALCULATION METHOD

2. Define how the parts share the bending.

Use a common-curvature, linear-elastic beam model. Both timber members and the steel plate extend over the full span and have their centroids at the same vertical level.

Adopted behaviour

- Small deflections; strong-axis bending; a pin and roller support; full-span uniform loading.
- Sufficient load transfer to maintain the same deflection and curvature in all three parts. Relative movement and connector slip are neglected.
- Timber is represented by its along-grain elastic modulus. Section properties remain constant along the span.

Scope of the outputs

- Gross, unperforated sections; no axial force or torsion.
- Immediate bending deflection only. Shear deformation, connection deformation, creep and moisture effects are excluded.
- Connections, load introduction, bearing, buckling, strength and detailing require separate assessment.

The neutral axis is at mid-depth because each component is vertically centred. Horizontal separation between the timber pieces adds no parallel-axis term for bending about this horizontal axis. An off-centre plate needs a different neutral-axis calculation.

Section geometry (1)

$$I_t = \frac{2b_t h_t^{\,3}}{12} \qquad I_s = \frac{t_s h_s^{\,3}}{12}$$

I_t includes both timber members. Units: mm^4.

Combined flexural rigidity (2)

$$K = E_t I_t + E_s I_s$$

K denotes combined EI, in $N{\cdot}mm^2$.

Peak moment and each support reaction (3)

$$M = \frac{wL^2}{8} \qquad R = \frac{wL}{2}$$

N·mm and N, with w in N/mm.

Steel and timber moment shares (4)

$$\eta_s = \frac{E_s I_s}{K} \qquad \eta_t = 1 - \eta_s$$

$M_s = \eta_s M$; $M_t = \eta_t M$ for the timber pair.

Extreme-fibre bending stress magnitudes (5)

$$\sigma_t = \frac{E_t M h_t}{2K} \qquad \sigma_s = \frac{E_s M h_s}{2K}$$

MPa; compression above N.A., tension below.

Midspan bending deflection and ratio (6)

$$\delta_b = \frac{5wL^4}{384K} \qquad u_\delta = \frac{\delta_b}{L/300}$$

Demo deflection PASS when $u_\delta \le 1$.

Derivation. A common curvature κ gives each component a moment $E_i I_i \kappa$. Adding the component moments gives $M = K\kappa$ and the shares above. Integrating the simply supported uniform-load moment diagram twice gives Equation (6). Mechanics background: references [1] and [2], page 4.

Units. Use mm, N and MPa throughout; 1 kN/m = 1 N/mm, 1 MPa = 1 N/mm^2, and 1 kN·m = 10^6 N·mm. Require positive dimensions, moduli and limit divisor, $w \ge 0$, and $0 < h_s \le h_t$.

REFERENCE CASE FB-03

3. Work through one beam.

Two 45 × 300 mm timber members and one centred 10 × 260 mm steel plate. Use the material moduli, 4,800 mm span and 10.0 kN/m total service load from page 1.

01 Calculate each stiffness contribution

$$I_t = 2 \times 45 \times 300^3 / 12 = 202{,}500{,}000 \text{ mm}^4$$

$$I_s = 10 \times 260^3 / 12 = 14{,}646{,}666.67 \text{ mm}^4$$

$$E_tI_t = 2.025 \times 10^{12} \text{ N·mm}^2$$

$$E_sI_s = 2.929333 \times 10^{12} \text{ N·mm}^2$$

$$\mathbf{K = 4.954333 \times 10^{12} \text{ N·mm}^2}$$

02 Calculate actions and allocate the moment

$$M = 10 \times 4{,}800^2 / 8 = 28{,}800{,}000 \text{ N·mm} = 28.80 \text{ kN·m}$$

$$R = 10 \times 4{,}800 / 2 = 24{,}000 \text{ N} = 24.00 \text{ kN per support}$$

$$\eta_s = 2.929333 / 4.954333 = 0.591267$$

$$M_s = 17.028 \text{ kN·m}; \quad M_t = 11.772 \text{ kN·m}$$

The timber pair takes the remaining 40.87%; each identical timber member takes half of that pair moment. This is bending-moment allocation, not a bolt-force calculation.

03 Calculate the elastic stress magnitudes

$$\sigma_t = 10{,}000 \times 28.8 \times 10^6 \times 150 / K = \mathbf{8.72 \text{ MPa}}$$

$$\sigma_s = 200{,}000 \times 28.8 \times 10^6 \times 130 / K = \mathbf{151.14 \text{ MPa}}$$

These are calculated demands. No material strength or stress-capacity pass/fail is assigned.

04 Compare bending deflection with the adopted target

$$\delta_b = 5 \times 10 \times 4{,}800^4 / (384 \times K) = \mathbf{13.95 \text{ mm}}$$

$$\delta_{lim} = 4{,}800 / 300 = 16.00 \text{ mm}$$

$$u_\delta = 13.95 / 16.00 = \mathbf{0.872 \leq 1}$$

59.13%	13.95 mm	PASS
steel bending-moment share	immediate bending deflection	adopted deflection test only

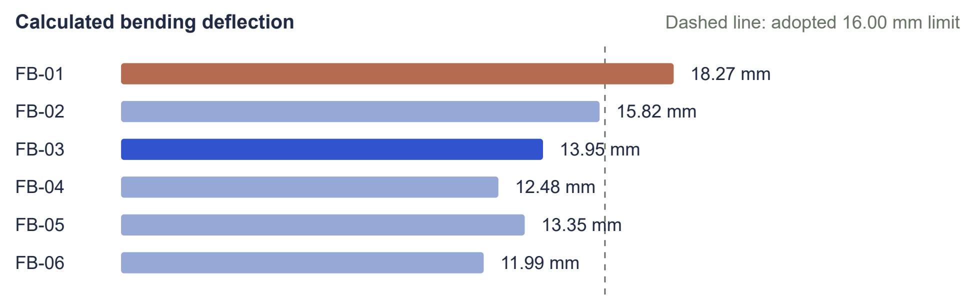
Use unrounded values in the calculation. Displayed values are rounded. A pass here does not establish total serviceability or member adequacy; the effects and checks excluded on page 2 remain outstanding.

SIX OPTIONS / ONE CALCULATION

4. Compare width and plate thickness.

Keep both depths, span, moduli, total service load and deflection target at their page-1 values. FB-03 is the worked reference.

Case	b_t mm	t_s mm	K 10^{12} N·mm^2	Steel share	δ_b mm	u_δ	Demo deflection
FB-01	45	6	3.783	46.5%	18.27	1.142	FAIL
FB-02	45	8	4.368	53.6%	15.82	0.989	PASS
FB-03	45	10	4.954	59.1%	13.95	0.872	PASS
FB-04	45	12	5.540	63.4%	12.48	0.780	PASS
FB-05	63	8	5.178	45.3%	13.35	0.834	PASS
FB-06	63	10	5.764	50.8%	11.99	0.749	PASS



> **What changes?** With the timber width fixed at 45 mm, increasing the plate from 6 to 10 mm reduces bending deflection from 18.27 to 13.95 mm. FB-02 clears the chosen limit by only 0.18 mm; excluded deformation effects could change that result. Wider timber also increases stiffness and reduces the steel moment share.

Source and data provenance

Fontana created the project scenario, diagrams, wording, input schedule and numerical comparison in this note. The material moduli and L/300 target are demonstration assumptions, not values assigned to a named product or grade. No strength tables, supplier span tables or standard extracts are reproduced.

[1] L. Bucciarelli, MIT OpenCourseWare. Stresses: Beams in Bending, section 9.5, printed pp. 251–254 (PDF pp. 17–20). Mechanics background for compatible curvature, elastic stress and summed flexural rigidity.

[2] M. T. Lo Ricco, M. O. Amini and D. R. Rammer (2021). Wood Handbook, Chapter 9: Structural Analysis Equations, FPL-GTR-282, pp. 9–2 to 9–4. Equation (9–2) and Table 9–1 distinguish bending and shear deflection; the simply supported full-span uniform-load bending coefficient is 5/384. Creep is discussed separately. This note retains only immediate bending deformation.

Companion data. The six input rows and calculated reference outputs are supplied separately as flitch-beam-cases.csv and flitch-beam-expected.csv. Their column headers include units.